

Viability and resilience in the dynamics of language competition

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IFISC



Language dynamics: What is this about?

Studies of the dynamics of human language can be divided into three main groups, depending on their specific point of view:

▶ *Language Evolution*: language as a complex system itself. Study of language structure dynamics and human history.

subject of study: language

▶ *Language cognition*: studies languages acquisition. Mainly centered in the study of human brain learning processes and genetics.

subject of study : person

▶ *Language competition*: dynamics of language use and learning due to social interactions, modeled in a network of social interactions

subject of study : society

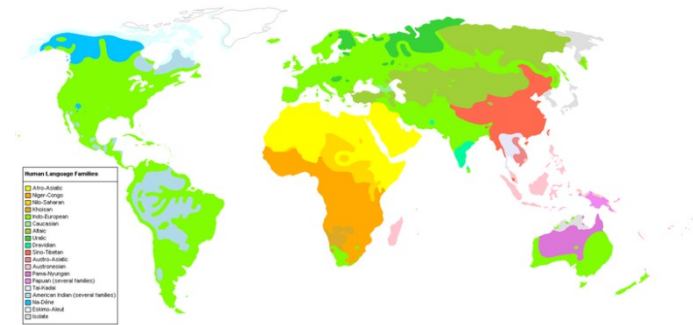
THIS WORK:

- Languages are given and fixed
- Examples: French-Flemish (Brussels,Belgium), Spanish-Catalan, Spanish-Quechua

Motivation: dynamics of language competition

Languages in the world today

- There exist around 6000 languages in the world.
- Over 50% of them are endangered (UNESCO).
- 4% of languages account for 96% of people.
- 25% of languages have less than 1000 speakers.



◆ D. Crystal. *Language Death* (Cambridge CUP 2000)

INFORMATION on languages in the world: ◆ <http://portal.unesco.org/culture>
 ◆ <http://www.ethnologue.com/>

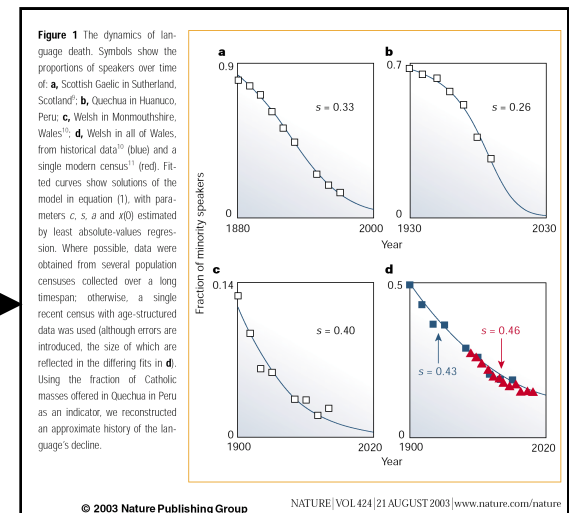
Language competition: *interdisciplinary research*

- "In order to capture the totality of this complex phenomenon it is essential not only to examine it from different disciplinary viewpoints but also to integrate these various viewpoints both at the theoretical and the methodological level in order to design interdisciplinary models. Because of the magnitude of the task no interdisciplinary team has yet made the attempt and succeeded. Studies of bilinguality and bilingualism are unidisciplinary, at most multidisciplinary, hardly ever interdisciplinary."
- Hamers, Josiane F. & Michel H.A. Blanc. 1989. *Bilinguality and Bilingualism*. Cambridge: C.U.P.

1st approach from complex systems

Abrams, Strogatz (2003). Nature 424, 900.

"Question: Extinction of endangered languages"



OUTLINE (*PATRES objectives*)

- i) **IBM:** presentation of the Individual Based Model for language competition (*case study in PATRES project*)
- ii) **MACROdescription:** Derive equations at the Macroscopic level in order to *understand* the global dynamics
- iii) **Viability & Resilience:** viable states: viability kernel
resilient states & policies

Individual Based Model for language competition

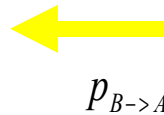
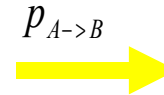
Agent-Based ABRAMS-STROGATZ model

Abrams, Strogatz (2003). Nature 424, 900.

$$p_{A \rightarrow B} = (1-s) \cdot (\sigma_B)^a$$

$$p_{B \rightarrow A} = s(\sigma_A)^a$$

Monolingual
A

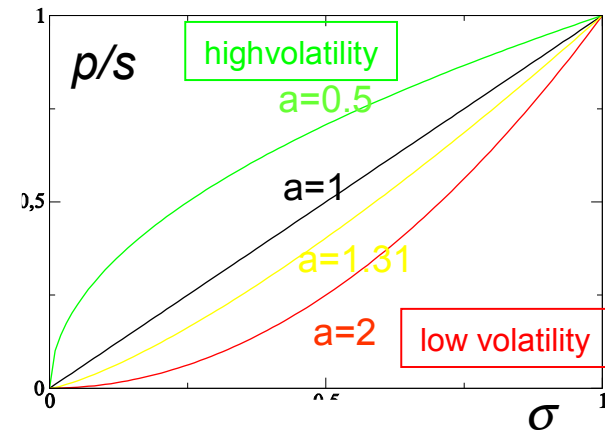


Monolingual
B

- Local density of speakers: $\sigma_{iA} = \frac{\# \text{ neighbours in state A}}{k_i}$ $\sigma_{iB} = \frac{\# \text{ neighbours in state B}}{k_i}$

s: prestige of language A ($s_B = 1-s$) language property
a: *volatility* (exponent) → determines the shape of $p_{A \rightarrow B}$
social dynamics property

AS-model for $a=1$ $s=1/2$ → VOTER model



MINETT-WANG MODEL

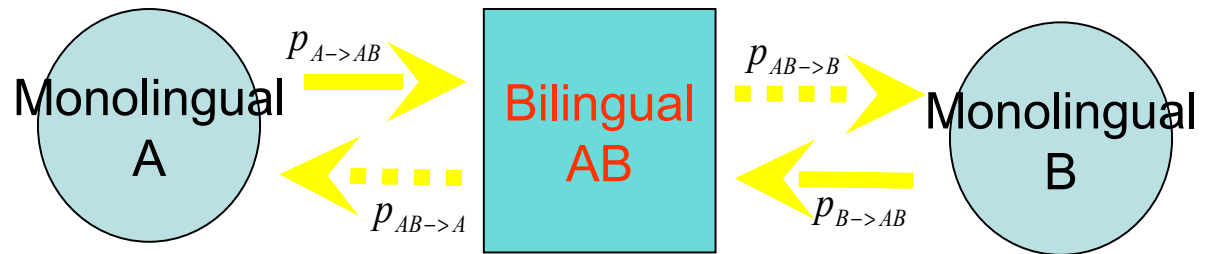
Wang, W. S-Y. and Minett, J. W, *Trends in Ecology and Evolution*, 20(5) 263 (2005) + *Lingua* 118(1) 19 (2008)

$$p_{A \rightarrow AB} = (1 - s) \cdot (\sigma_B)^a$$

$$p_{B \rightarrow AB} = s \cdot (\sigma_A)^a$$

$$p_{AB \rightarrow A} = s \cdot (1 - \sigma_B)^a$$

$$p_{AB \rightarrow B} = (1 - s) \cdot (1 - \sigma_A)^a$$



APPLET

FROM IBMS TO MACROSCOPIC DESCRIPTIONS

COMPLETELY CONNECTED NETWORK

→ population dynamics

$$\frac{dx}{dt} = x(1-x) [s x^{a-1} - (1-s)(1-x)^{a-1}]$$

x : global density of X-speakers
 y : global density of Y-speakers

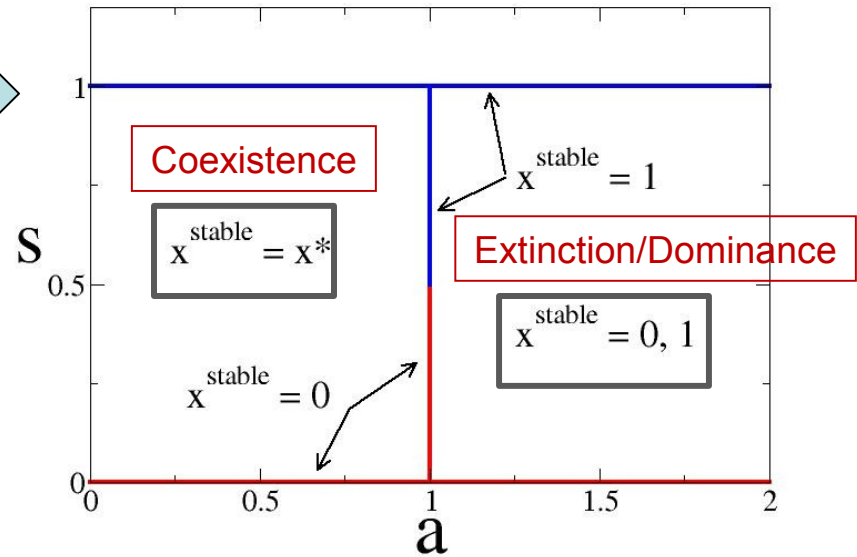
Stability analysis
 (stationary solutions)

$$x^s = \begin{cases} 0 & \text{y consensus} \\ 1 & \text{x consensus} \\ x^* = \frac{(1-s)^{1/(a-1)}}{(1-s)^{1/(a-1)} + s^{1/(a-1)}} & \text{x - y coexistence} \end{cases}$$

Stability diagram



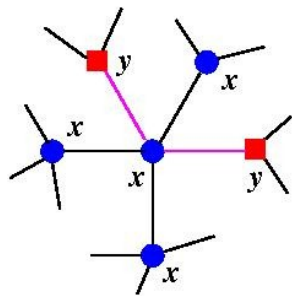
<u>COEXISTENCE</u>	$a < 1$
<u>EXTINCTION (DOMINANCE)</u>	$a > 1$



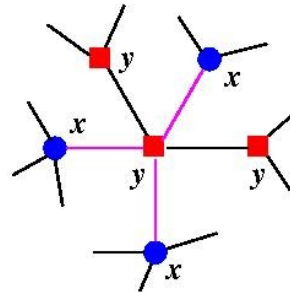
LOCAL EFFECTS i) RANDOM GRAPH

— n x-y links
 — μ-n x-x links

Each node connected (on average) to μ neighbors chosen at random.



$$P(x \rightarrow y) = (1-s) \left(\frac{n}{\mu}\right)^a$$



$$\Delta \rho = \frac{2(\mu - 2n)}{\mu N}$$

$$\Delta x = \frac{-I}{N}$$

a=1 case (*biased voter model*)

$$\frac{dx}{dt} \simeq \frac{(2s - 1)(\mu - 2)}{4[\mu + 2(s - 1)]} x(1 - x)$$

General case

$$\frac{dx}{dt} = 2 \xi \mu^{-a} x(1 - x) \sum_{n=0}^{\mu} n^a \frac{\mu!}{n! (\mu - n)!} \left\{ s x^{n-1} (1 - 2\xi x)^{\mu-n} - (1 - s)(1 - x)^{n-1} [1 - 2\xi(1 - x)]^{\mu-n} \right\}$$

Stability diagram

μ : local effects of finite # neighbors

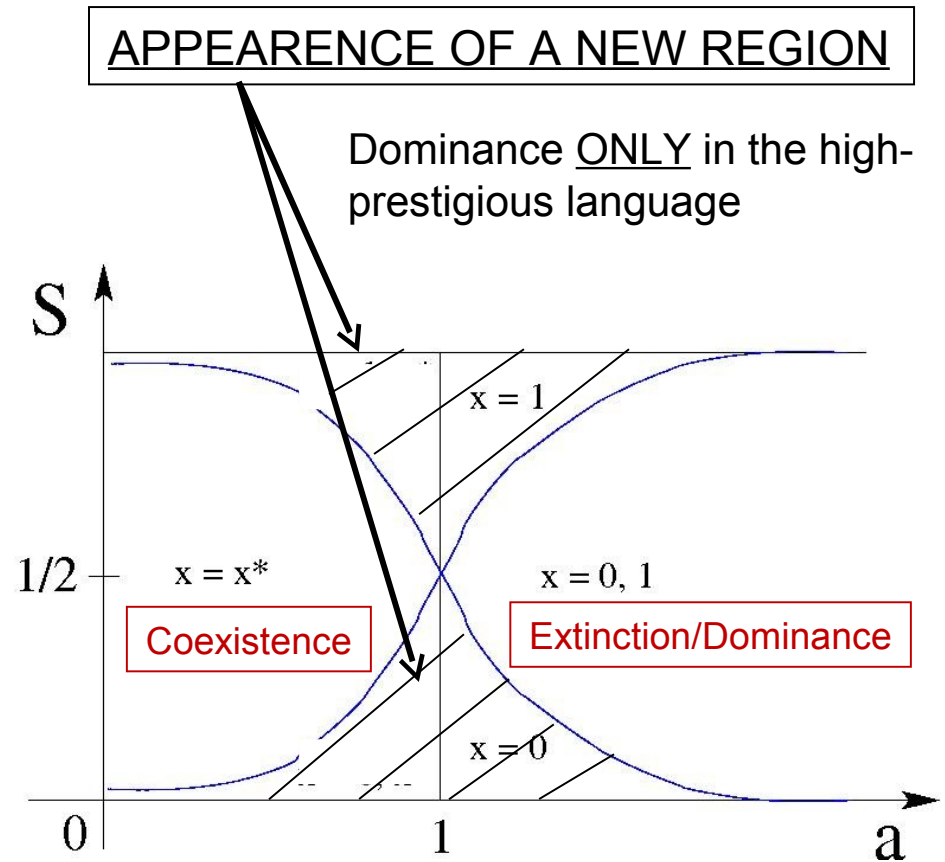
$\mu \rightarrow N^{-1}$ we recover mean field case

(COEXISTENCE $a < 1$)
(EXTINCTION $a > 1$)

$\mu \downarrow$ new region gets wider

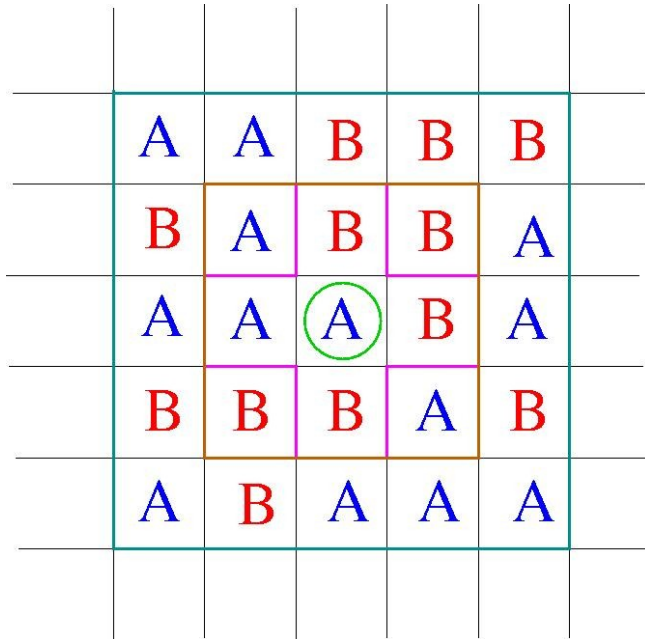
Coexistence gets more difficult!

Confirmed by numerical spreading experiments



LOCAL EFFECTS ii) 2D-REGULAR NETWORK

(case $s=1/2$)



A \rightarrow 1, B \rightarrow -1

ϕ_r = "language field" at site r .

$$-1 \leq \phi_r \leq 1$$

$$P(\mp \rightarrow \pm) = \sigma_{\pm}^q = \left(\frac{1 + \psi_r}{2} \right)^q$$

ψ_r = neighboring field

$$\psi_r = \sigma_+ - \sigma_- = 2\sigma_+ - 1$$

$$\psi_r = \phi_r + \Delta\phi_r$$

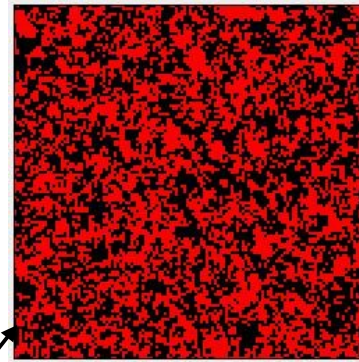
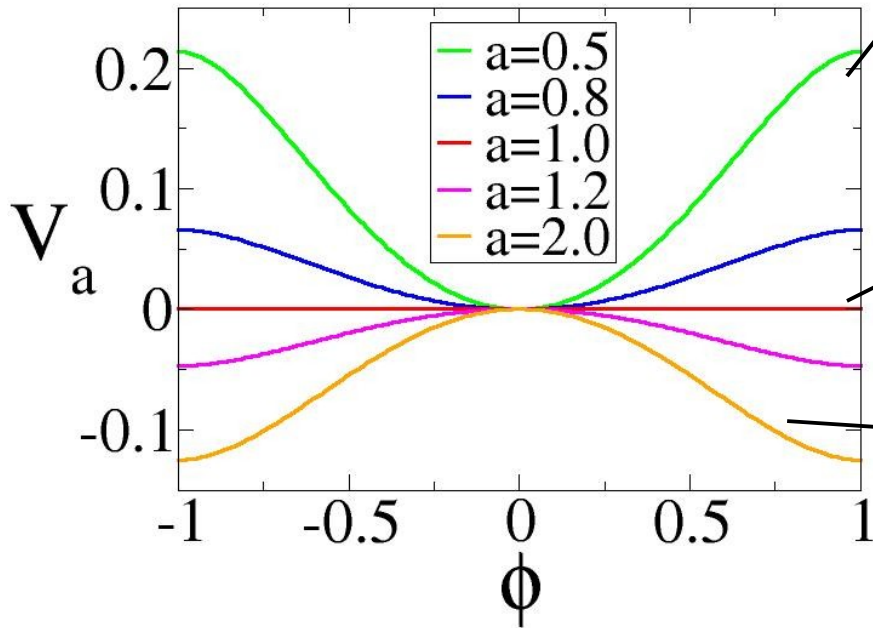
Equation for the evolution of ϕ_r :

$$\frac{\partial \phi_r(t)}{\partial t} = [1 - \phi_r(t)] P(- \rightarrow +) - [1 + \phi_r(t)] P(+ \rightarrow -) + \eta_r(t)$$

$$\frac{\partial \phi}{\partial t} = D \Delta \phi - \frac{\partial V_a(\phi)}{\partial \phi}$$

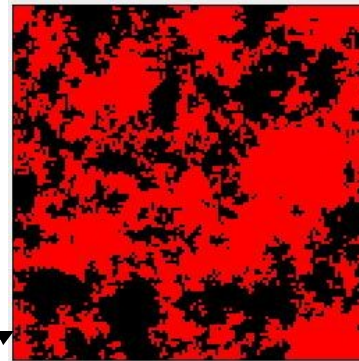
Time-dependent Ginzburg-Landau Equation with potential $V_a(\phi)$.

$$V_a(\phi) = -\frac{(a-1)}{3 \times 2^a} \left\{ 3\phi^2 + [(a-2)(a-3) - 6] \frac{\phi^4}{4} - (a-2)(a-3) \frac{\phi^6}{6} \right\}$$

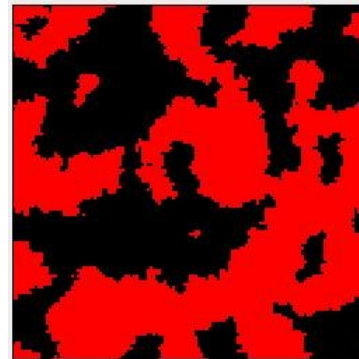


■ A-speakers
■ B-speakers

$a = 0.5$
Disordered active state.



$a = 1.0$
Ordering without surface tension.



$a = 2.0$
Ordering by surface tension.

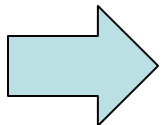


VIABILITY AND RESILIENCE

VIABILITY AND RESILIENCE. Motivation

Contrary to the model's stark prediction, bilingual societies do, in fact, exist.... The example of Quebec French demonstrates that language decline can be slowed by strategies such as policy-making, education and advertising, in essence increasing an endangered language's status. An extension to [the model] that incorporates such control on s through active feedback does indeed show stabilization of a bilingual fixed point.

Abrams, Strogatz (2003). Nature 424, 900



Viability theory: provides theoretical concepts and practical tools to maintain a dynamical system inside a given set of a priori desired states

→ language coexistence

LANGUAGE VIABILITY. Abrams-Strogatz model

Viability constraint set: given set of a priori desired states
 → safe language coexistence

Viability kernel: set of all *viable states* (i.e., states such that there exist at least one control function that maintains indefinitely the system inside the viability constraint set)

The language viability problem: → *completely connected network*

$$\frac{d\Sigma_A}{dt} = (1 - \Sigma_A)\Sigma_A(\Sigma_A^{a-1}s - (1 - \Sigma_A)^{a-1}(1 - s))$$

state variable → *density of speakers of language A*

$$\frac{ds}{dt} = u$$

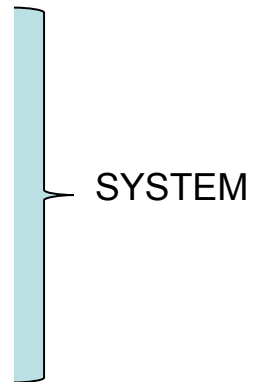
control variable → *time variation of the prestige (government action)*

$$u \in [-c, c]; c \in [0, 1]$$

restrictions on control → not any action is possible!! (c=0.1)

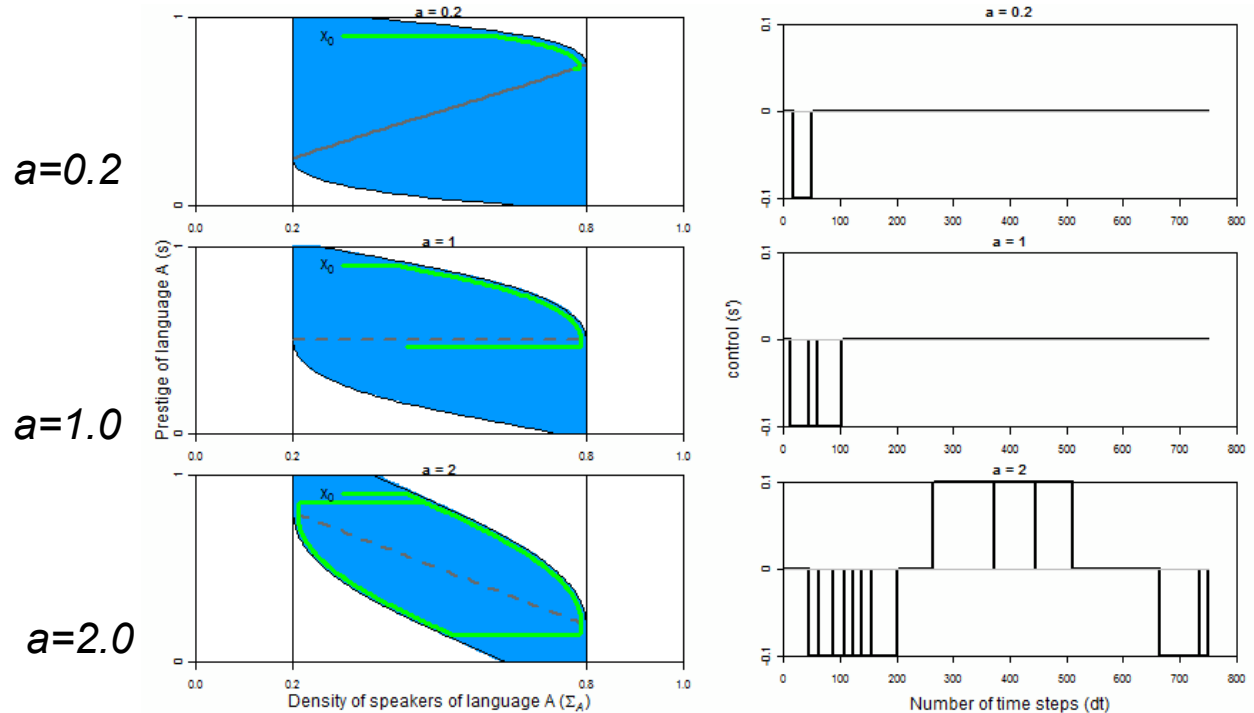
$$K = [\Sigma_A, \overline{\Sigma_A}] \times [0, 1]$$

constraint set. We fix: $\underline{\Sigma_A} = 0.2; \overline{\Sigma_A} = 0.8$



The viability kernel

→ $\text{Viab}(K)$



- Dependence on the volatility, a : $a \uparrow \rightarrow$ *viability kernel shrinks!*

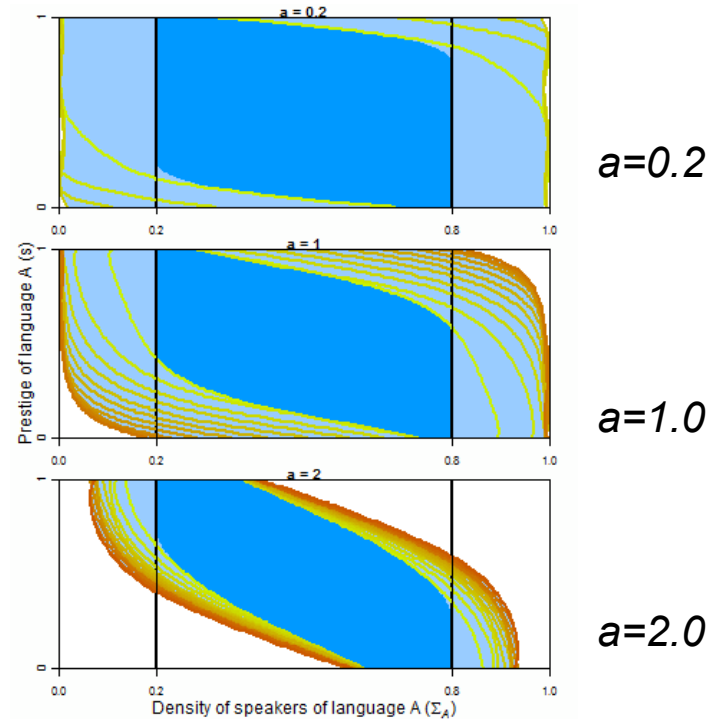
as agents have larger inertia to change, control is less effective

remember: $[a < 1 \rightarrow$ coexistence STABLE $a > 1$ coexistence UNSTABLE]

- *Heavy viable trajectories* → apply control iff trajectory will move outside viability kernel

LANGUAGE RESILIENCE

Dark blue	Viab(K)
Light blue	Resilient
White	Non-resilient
iso-cost lines (in color)	



Resilience: capacity of a system to restore its properties of interest lost after a perturbation
 → language recovery after being in an endangered situation

Computation of resilience values → $resilience \propto \frac{1}{cost}$

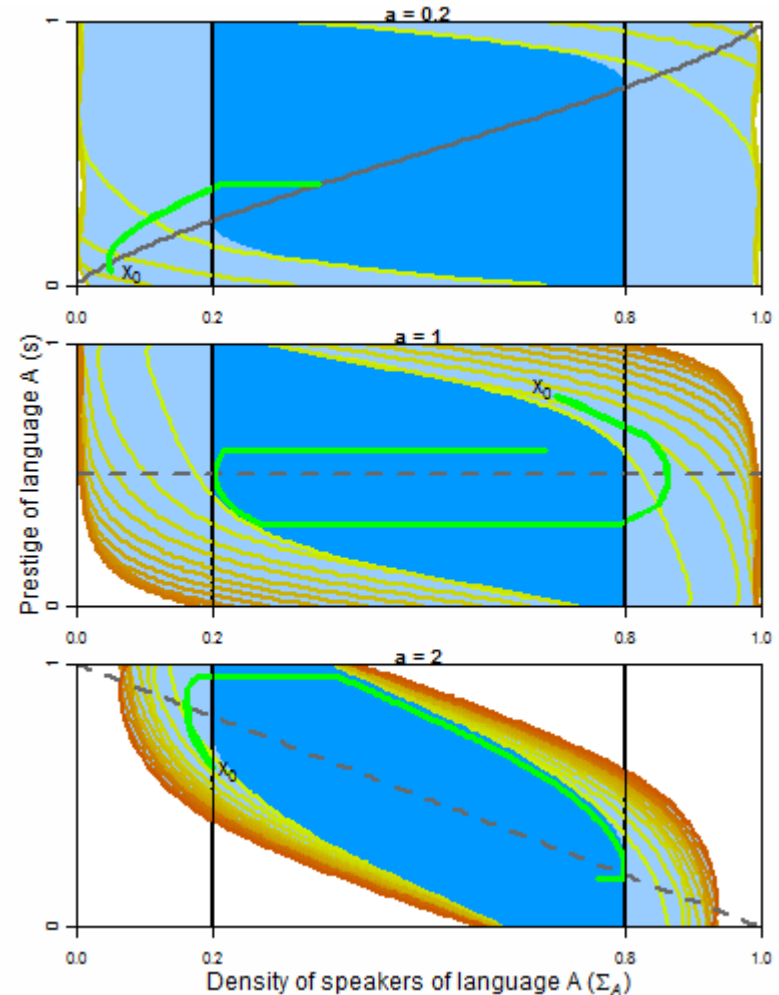
Cost function:
$$C(x) = \min_{x(.)} \left(\int_0^{+\infty} \chi_{x(t) \notin Viab(K)} dt + c_2 d(x(t), K) \chi_{x(t) \notin K} dt \right)$$

Determining action policies to restore viability at minimum cost

a) Consider a state with finite resilience

b) Choose action policy that decreases the cost at each time step

c) When reaching $Viab(K)$ use heavy control procedure (ensure viability)



CONCLUSIONS

→ *language competition case study*

■ Individual Based Model: *Abrams-Strogatz model*

state variable: language of speakers

parameters of the model:

prestige, s
volatility, a

■ Macroscopic description

- Compl. Connected: stability diagram (SD) → $a < 1$ (coexistence) $a > 1$ (extinction/dominance)

- Local effects (i) Random graphs (finite # neighbors) → coexistence is reduced (*new region*)
→ prestige becomes more determinant

- Local effects (ii) 2d regular network Analysis of the dynamics at the linguistic borders
→ effects of volatility on *stability* and *domain growth*

■ Viability and resilience

- Viability: computation of viable policies to keep language coexistence. $Viab(K)$

$Viab(K)$: shrinks with increasing *volatility*

- Resilience: determination of resilient scenarios, and the corresponding action policies that can drive back an endangered language to a situation of safe coexistence

CONCLUSIONS II

Within the assumptions and limited framework of current models for competing languages:

Interpretation: stability depending on volatility:

- Society in which agents have more inertia to change their language use (*low volatility* $a > 1$)
 - might be due to cultural/ethnic attachment to language
 - extinction/dominance

- Society in which agents more easily change their language use (*high volatility* $a < 1$)
 - coexistence

MINETT-WANG MODEL

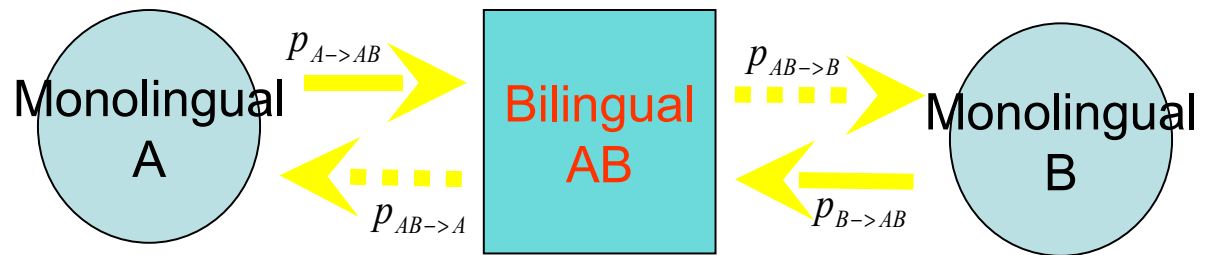
Wang, W. S-Y. and Minett, J. W, Trends in Ecology and Evolution, 20(5) 263 (2005) + Lingua 118(1) 19 (2008)

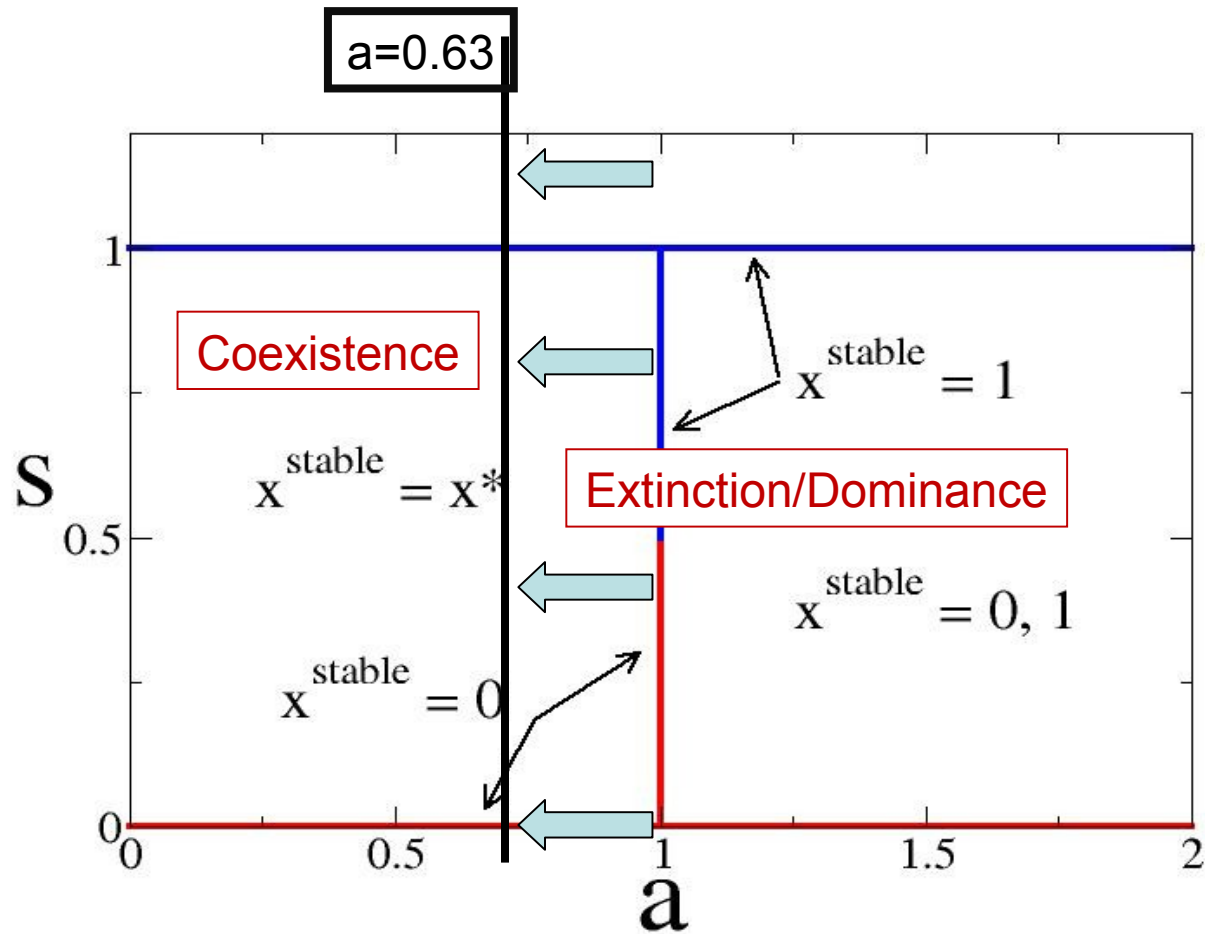
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Minett-Wang model: bilingual agents in use \rightarrow reduce coexistence !! $a_c \rightarrow 0.63$

More volatile agents are needed to ensure language coexistence

THANK YOU FOR YOUR ATTENTION!

Other related publications:

X. Castelló, V. Eguíluz and M. San Miguel.

+ Dietrich Stauffer,
+ Lucía Loureiro Porto

+ R. Toivonen, J. Saramäki, K. Kaski
+ R. Toivonen, J. Saramäki, K. Kaski