PATRES
PATTERN RESILIENCE
EU FRAMEWORK 6 NEST PROJECT

Bridging the gap between structurally realistic models and viability theory in savanna ecosystems

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PATRES: Savanna case study

Overview:

1) Start with a detailed, site-specific savanna model

2) Identify essential pattern dynamics of the model

3) Build a low dimensional, mathematically tractable approx. capturing key pattern dynamics

4) Apply viability theory to approximate model

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Savannas

Defined by:

1) Continuous grass layer
2) Discontinuous tree layer

Cover ~20% of Earth's terrestrial surface area

Harbor considerable biodiversity

Economically significant as grazing lands

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Current threats to savannas

Human population growth

Global climate change

Changing land use patterns

Overexploitation by humans
-Specifically overgrazing

Bush encroachment

Savanna management

Must balance system integrity against economic goals

Price of failure is high. Bush encroached savannas effectively useless for grazing

Manage by manipulating fire and/or grazing regimes

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“Structurally realistic” savanna models

Relatively complex models used to integrate knowledge on particular savannas

Typically stochastic. Include many parameters and variables

Often reproduce key features of focal site

Seldom used for management purposes

The Jeltsch model

Developed by Jeltsch and colleagues in 1990's

Focuses on a South African savanna

Integrates 30+ years of detailed empirical data from this site

Widely cited and still widely used

Jeltsch et al. 1996; 1997a, b; 1998;1999

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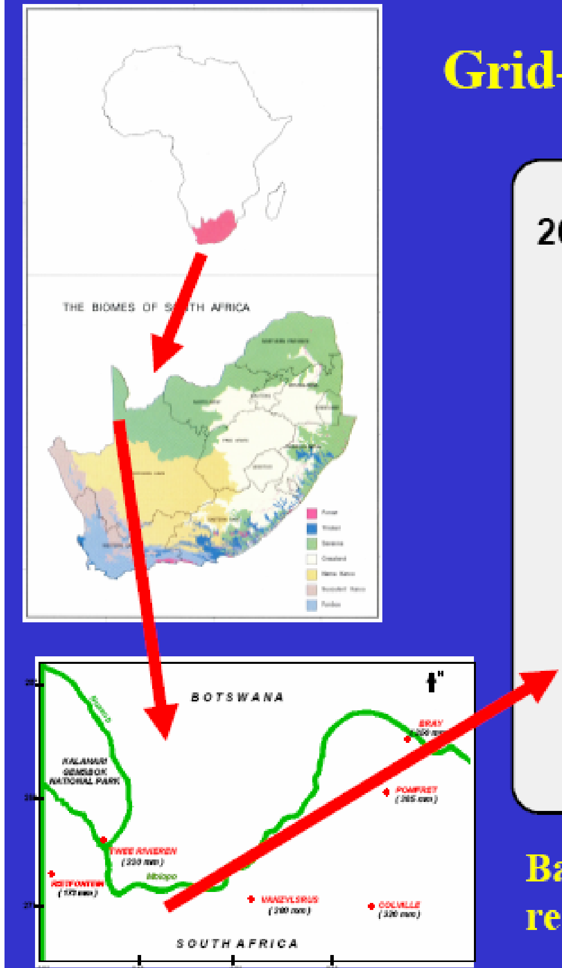
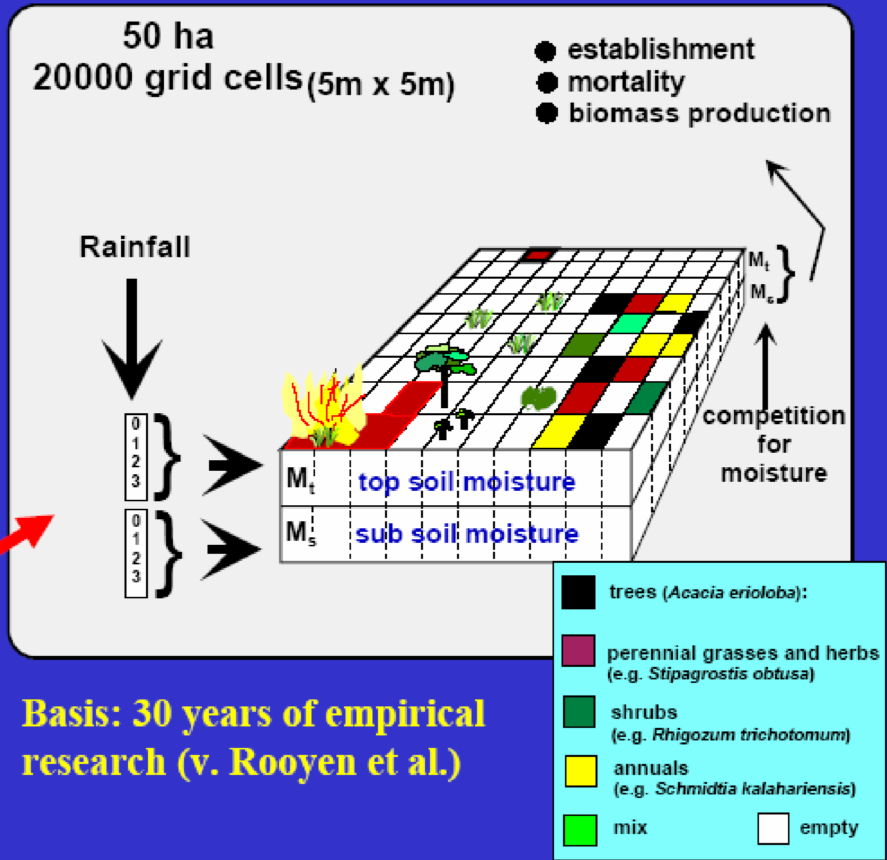
Grid-based savanna model





Basis: 30 years of empirical research (v. Rooyen et al.)

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Complex model, but complex dynamics?

Dynamics of macroscopic state variables might not be complicated

This appears to be the case for the Jeltsch model

Idea: Use the relatively simple dynamics of a few key patterns to reduce model dimensionality

Targets of analysis

Focus on trees:

1) Population density time series $\left(\rho_1 = \frac{\# trees}{\# sites} \right)$

2) Distance-dependent spatial pattern (*g* statistic)

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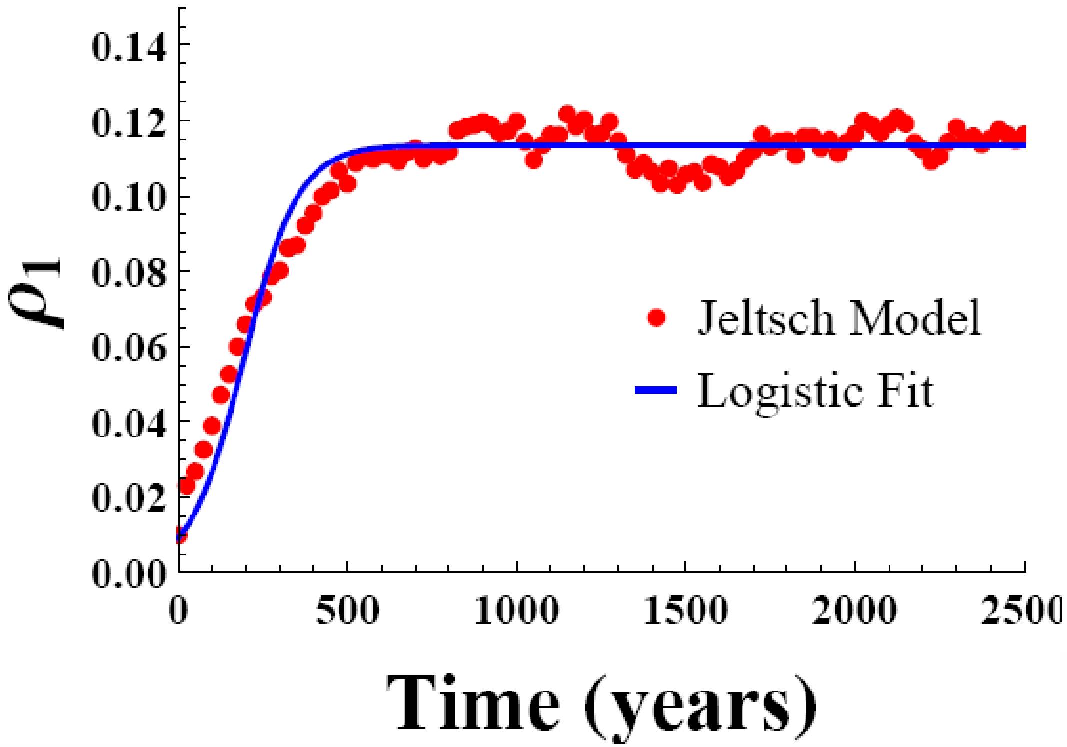
Logistic-like population growth



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Point-pattern analysis

Normalized pair correlation statistic

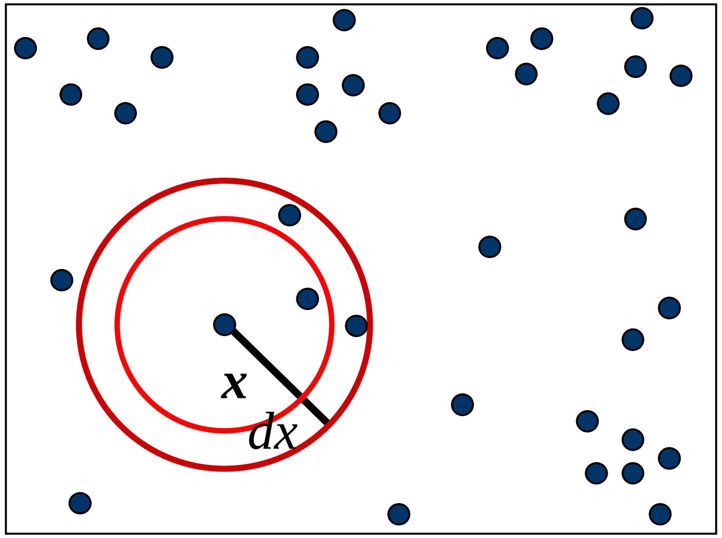
Stoyan & Stoyan 1994; Condit et al. 2000

$$g_x = \frac{\sum N_x}{K \sum A_x}$$

g=1 ⟶ Random

g>1 ⟶ Clustered

g<1 ⟶ Regular



Where K is the density of the point pattern, and N_x A_x are the # of neighbors and area of the annulus, respectively, at dist. x.

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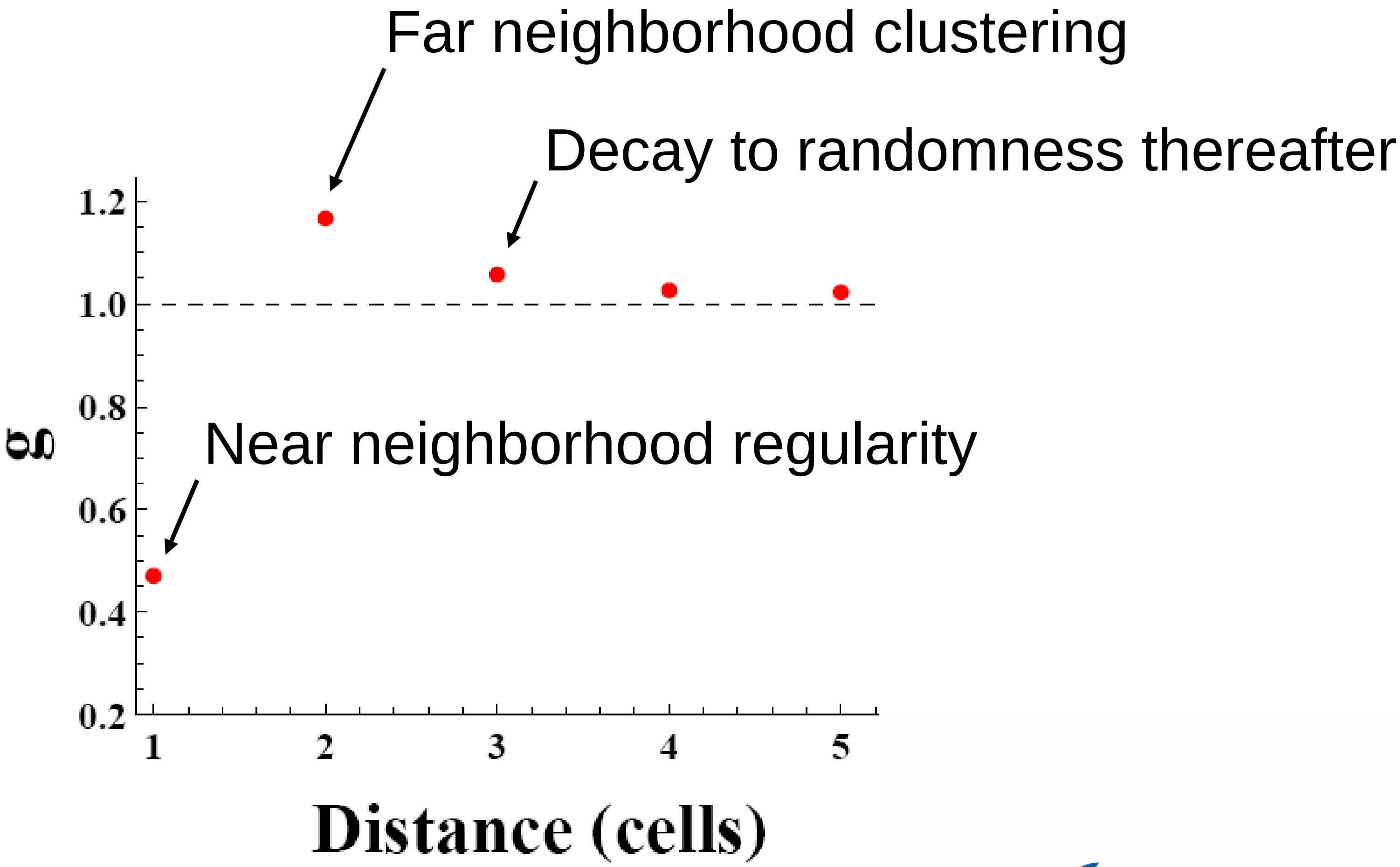
Strong signatures of competition and short dispersal



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Key patterns

Logistic-like population growth

Spatial pattern:

1) Near neighborhood regularity
2) Far neigh. clustering
3) Decays to randomness thereafter

Suggests a ***spatial logistic model*** with 2 interaction scales

Model overview

Square lattice, periodic boundary conds., two states:
tree (1) or grass (0)

Prop. of sites in state 1 is ρ_1 and in state 0 is $\rho_0 = 1 - \rho_1$

Model is an extension of the contact process

Tree dispersal & competition occur within defined (Moore) neighborhoods

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Interaction neighborhoods

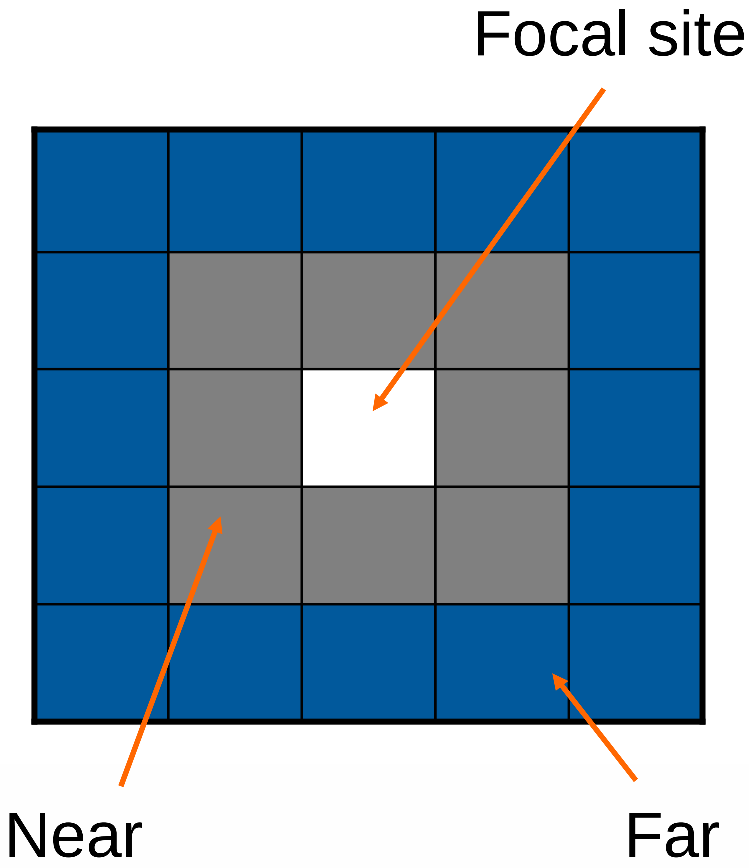
Define two neighborhoods—
”near” and “far”

Assume:

1) establishment comp. occurs only over near neigh.

2) birth occurs in both near & far neighborhoods

3) Neighborhood status is symmetric



Disp. scale > Comp. scale

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Birth

Trees reproduce at constant rate *b*

Each site within birth (*n* + *f*) neighborhood of a tree receives offspring at rate:

$$\beta = \frac{b}{z_n + z_f}$$

If an seed lands on a tree occupied site, nothing happens

If it lands on a grass occupied site, it has a chance to establish

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Establishment

Given birth, the seed establishes with probability

$$P_E = P_C^{Surv} P_F^{Surv}$$

where:

$$P_C^{Surv} = e^{-\delta C}$$ and $$P_F^{Surv} = 1$$ (for now)

Moment approximations: Density

$q_{1/0}$ and $\tilde{q}_{1/0}$ are the near & far neighborhood local densities

Pair:

$$\frac{d\rho_1}{dt} = \beta\left(z_n\, q_{1/0} + z_f\, \tilde{q}_{1/0}\right)\left(1-\rho_1\right) P_F^{Surv}\, e^{-\delta\, z_n\, q_{1/0}} - \rho_1$$

Mean field:

$$\frac{d\rho_1}{dt} = \beta\left(z_n\, \rho_1 + z_f\, \rho_1\right)\left(1-\rho_1\right) P_F^{Surv}\, e^{-\delta\, z_n\, \rho_1} - \rho_1$$

Multiscale pair approximation: Ellner 2001

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Pair Approx.: Pair correlations

Local dens. can be written in terms of pair and singlet probs.

$$q_{1/0} = \frac{\rho_1 - \rho_{11}}{1-\rho_1} \quad \& \quad \tilde{q}_{1/0} = \frac{\rho_1 - \tilde{\rho}_{11}}{1-\rho_1}$$

Each int. neighborhood requires a pair dens. eqn.

Near:

$$\frac{1}{2}\frac{d\rho_{11}}{dt} = \beta\left(1+(z_n-1)q_{1/0} + z_f\,\tilde{q}_{1/0}\right)\left(\rho_1 - \rho_{11}\right)P_F^{Surv}\,e^{-\delta\left[1+(z_n-1)q_{1/0}\right]} - \rho_{11}$$

Far:

$$\frac{1}{2}\frac{d\tilde{\rho}_{11}}{dt} = \beta\left(z_n\,q_{1/0} + 1 + (z_f-1)\tilde{q}_{1/0}\right)\left(\rho_1 - \tilde{\rho}_{11}\right)P_F^{Surv}e^{-\delta z_n q_{1/0}} - \tilde{\rho}_{11}$$

Full analysis of PA savanna model in Calabrese *et al.* (*In press, AmNat*)

Back to the Jeltsch model

We have an equation for ρ_1

G statistics can be derived from PA: $g_n = \frac{\rho_{11}}{\rho_1^2}$ $g_f = \frac{\tilde{\rho}_{11}}{\rho_1^2}$

PA has two free parameters: *b* and δ

They can be estimated by fitting the PA to the Jeltsch model

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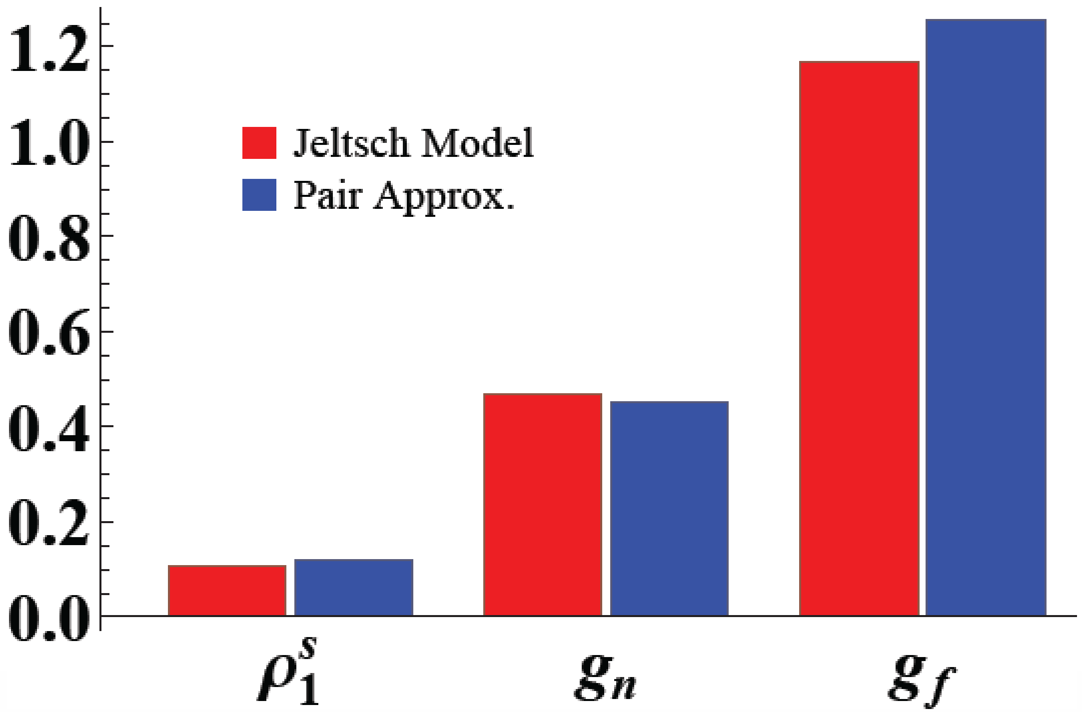
Fit PA to 3 patterns



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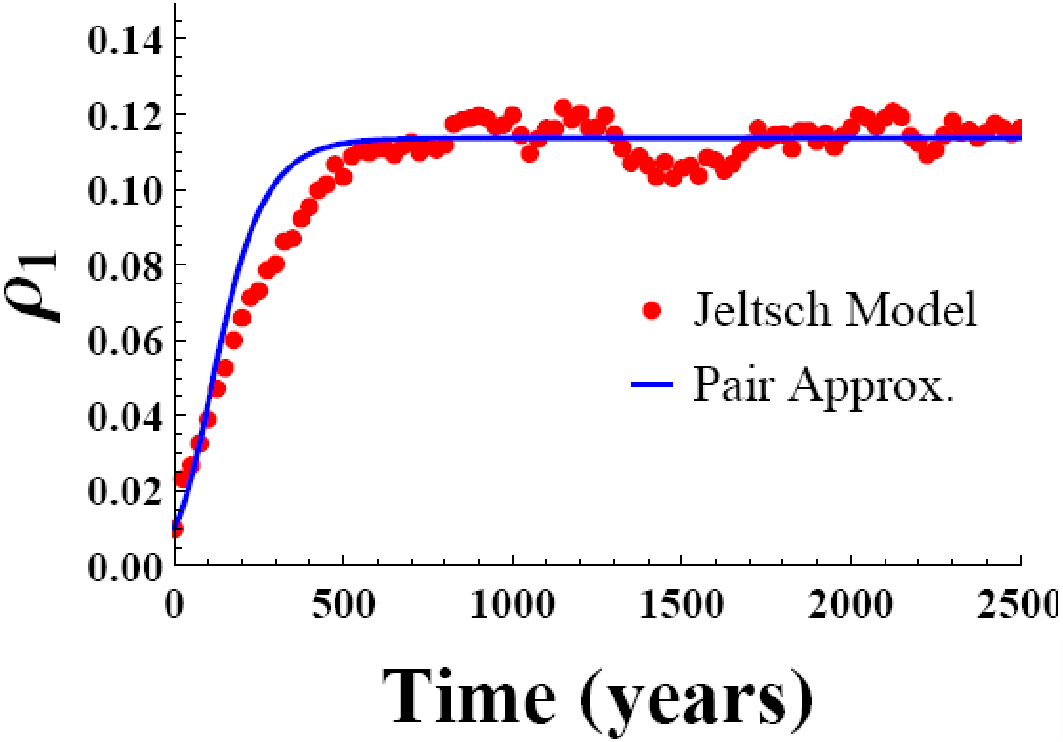
Transient dynamics



Adding fire

Negatively affects trees by killing primarily juveniles

Fire regimes can be manipulated to control trees

Direct: Prescribed burns
Indirect: Varying grazing pressure

Fire represents a key control action

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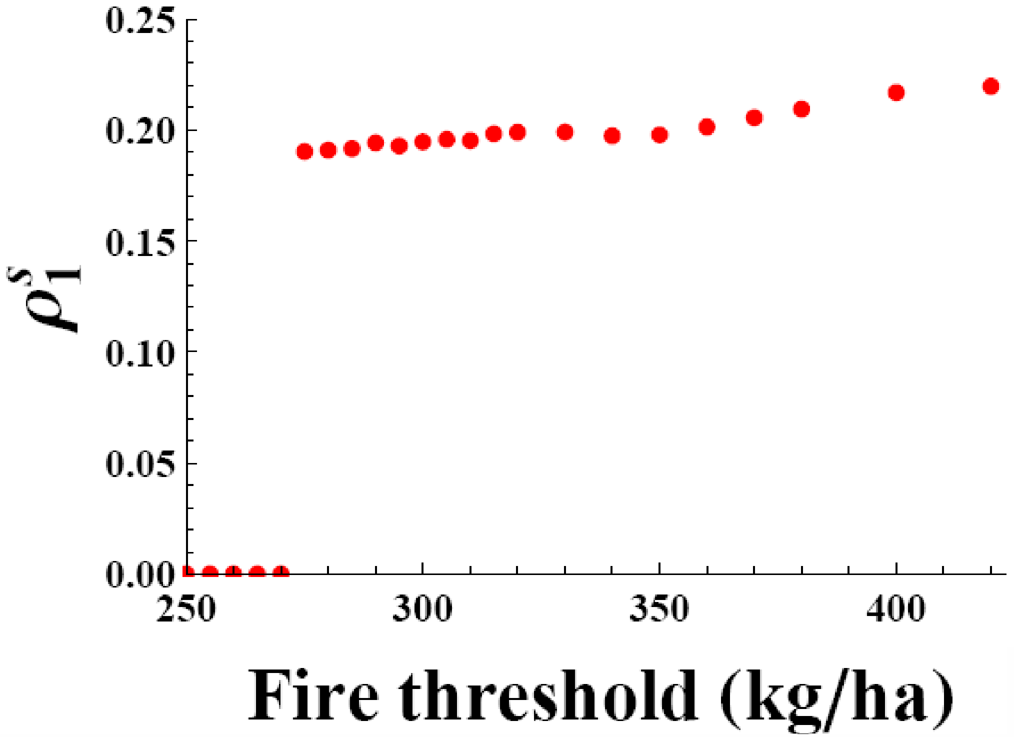
Fire in the Jeltsch model



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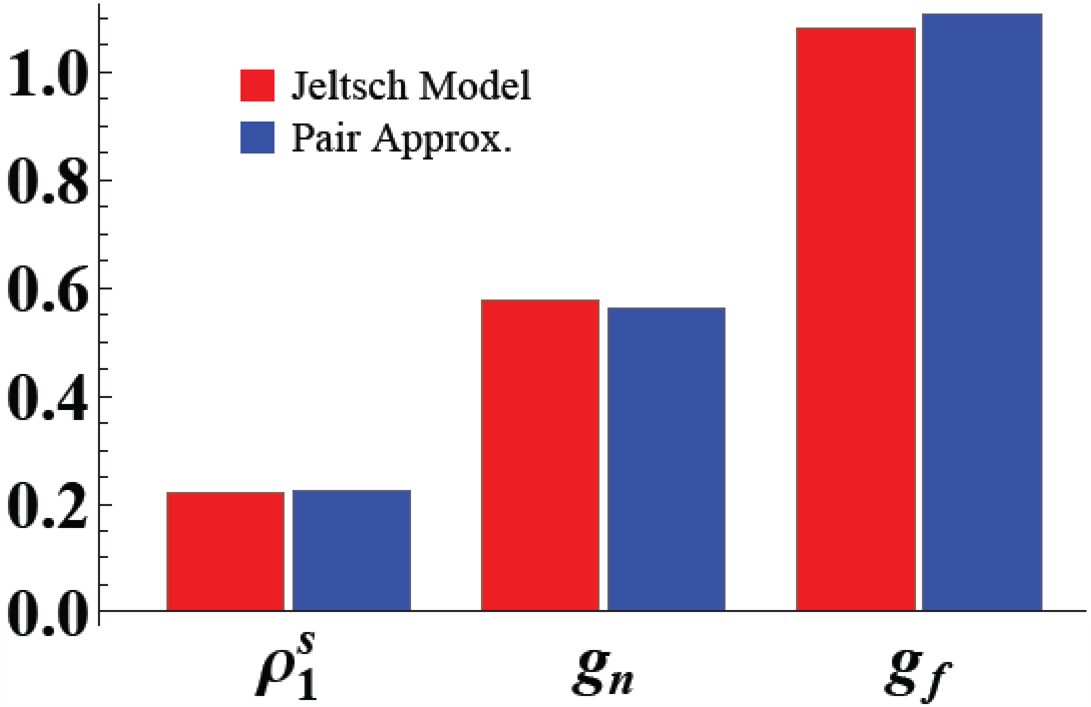
First fit competition model w/o fire

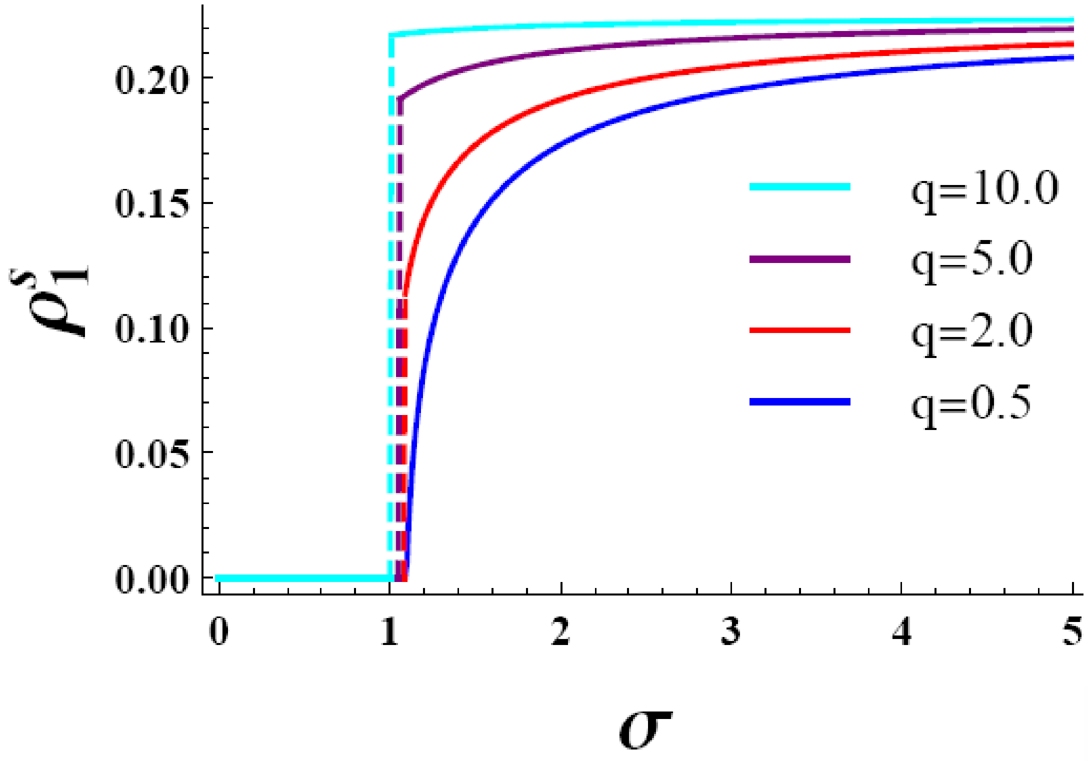


Fire in pair approx.

$$P_F^{Surv} = 1 - \frac{1}{\sigma}(1 - \rho_1)^q$$

σ affects fire freq.

q determines abruptness of trans.



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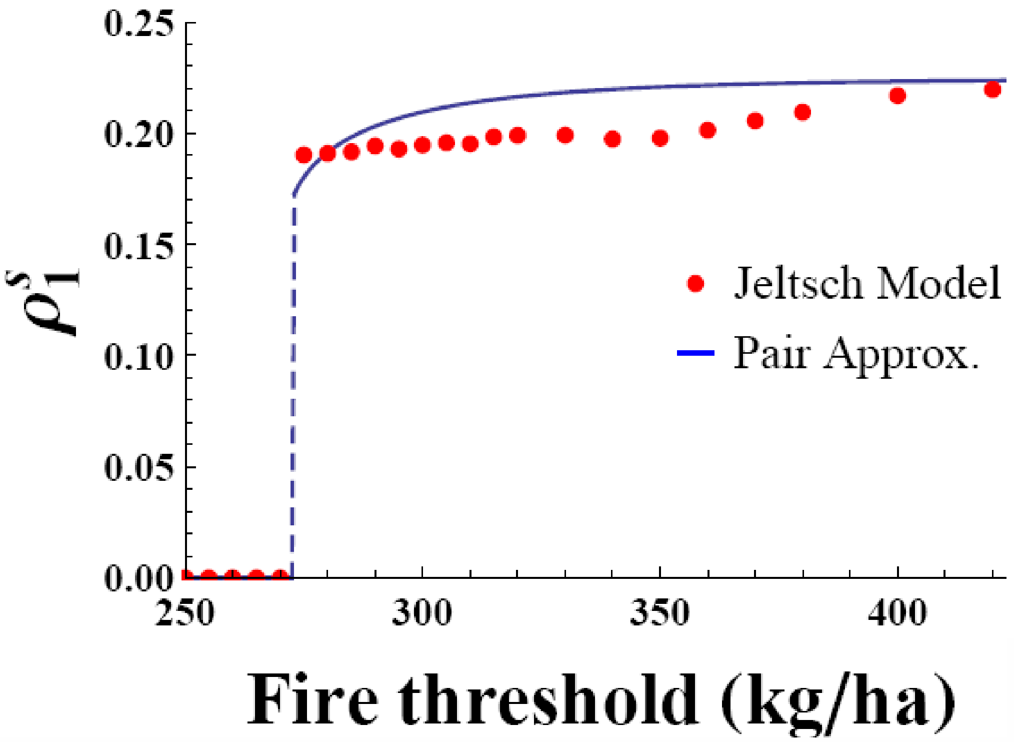
Comparing the transitions



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A viability problem based on MF approx.

$$\frac{d\rho_1}{dt} = be^{-\delta z_n \rho_1} \frac{\sigma}{\sigma + 1 - \rho_1}(\rho_1 - \rho_1^2) - \rho_1 = \phi(\rho_1, h)$$

where $\sigma = \hat{\sigma}/(1-h)$

Redefine system to include dynamics of control:

$$\rho_1(t+dt) = \rho_1(t) + \phi(\rho_1(t), h(t))dt$$
$$h(t+dt) = h(t) + u(t)dt$$

Subject to constraints:

$$\rho_{1Min} \le \rho_1(t) \le \rho_{1Max} \qquad |u(t)| \le h'_{Max}$$

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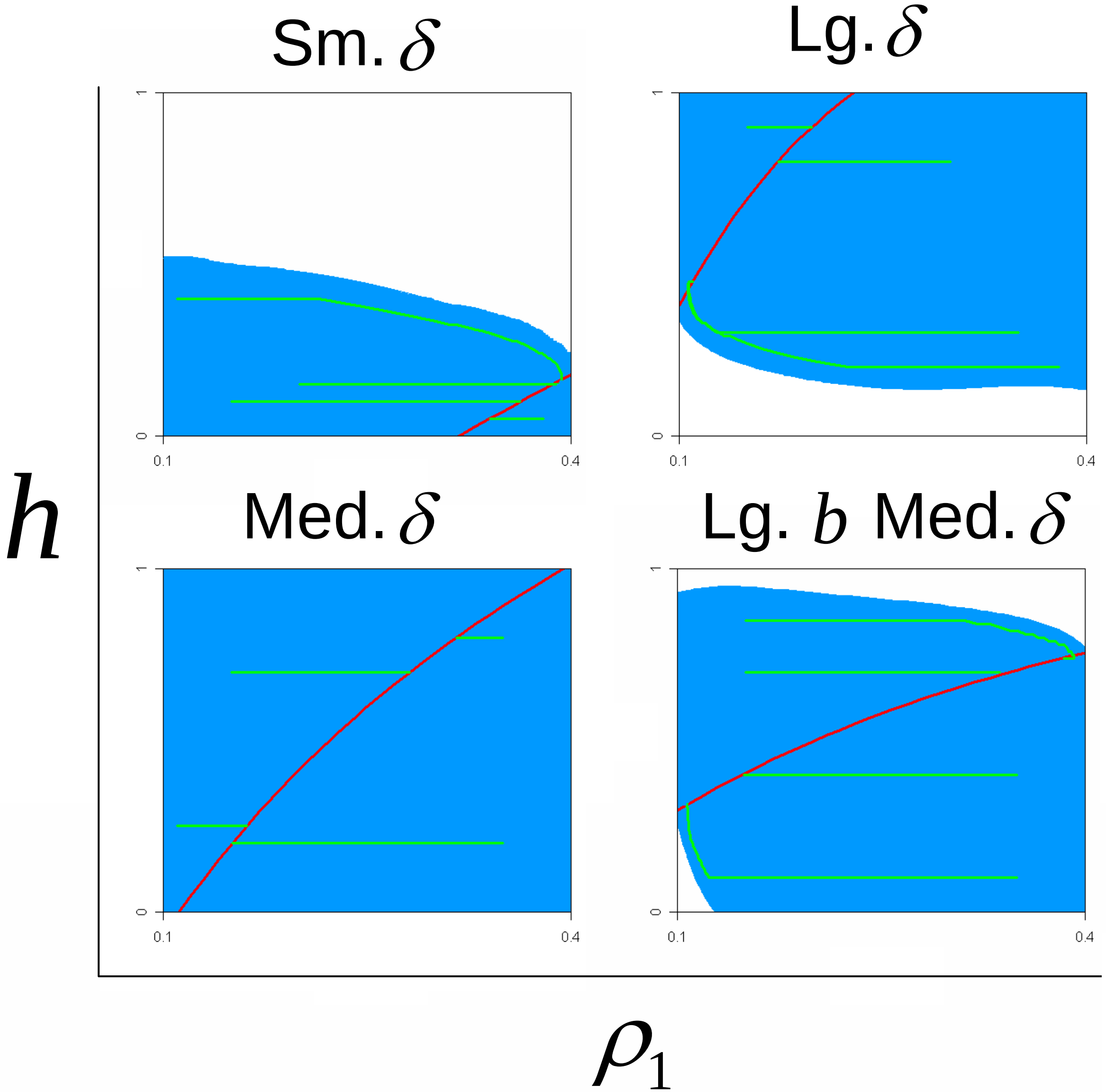
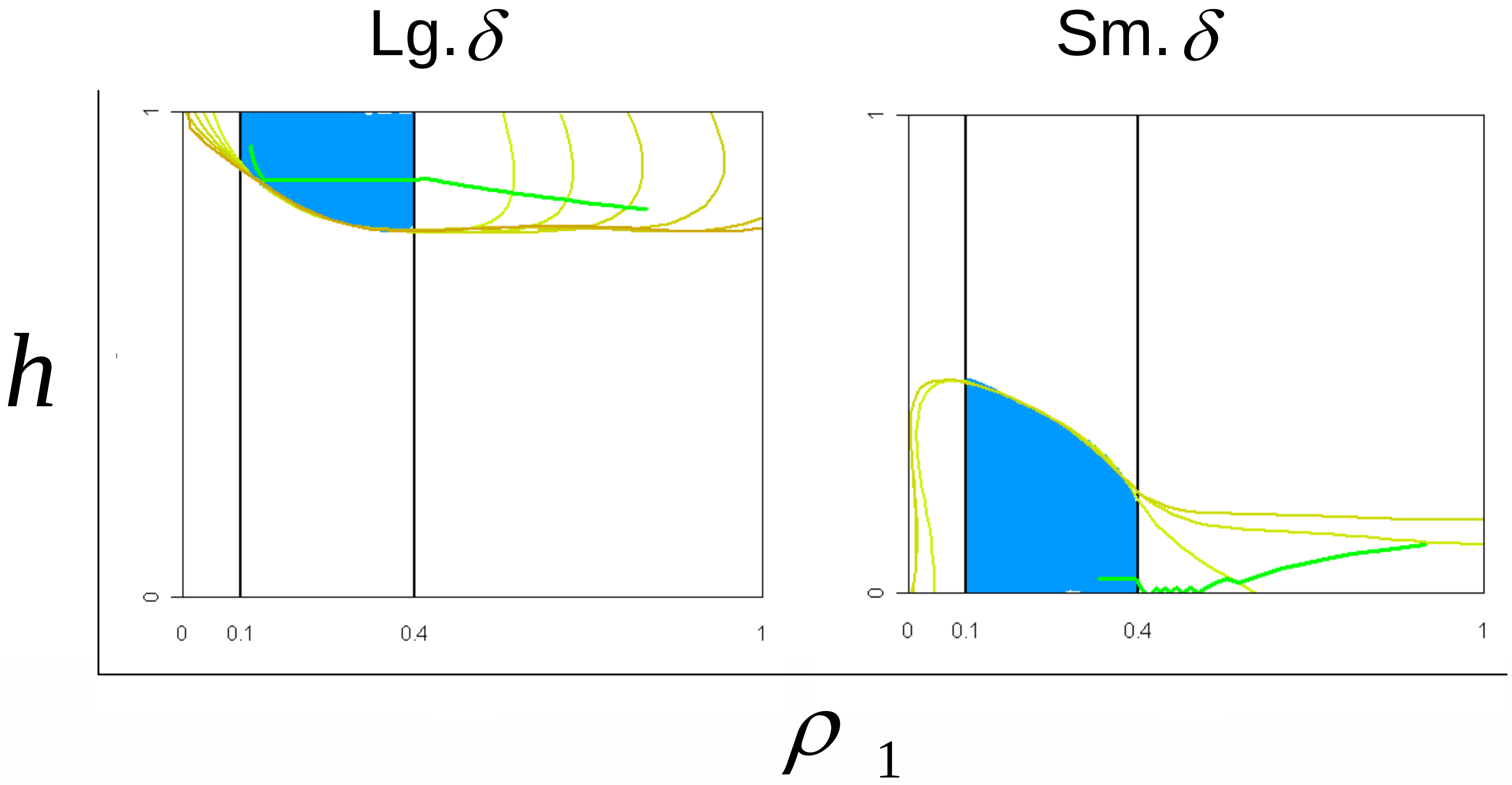
Viability kernels



Quantifying resilience



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Conclusions

Path from Jeltsch mod. to viab. analysis is long & winding

Moment equations can help simplify complex models

Mapping between control parameters in the two models needs further study

Next step is to apply a control policy identified using the approx. mod. to the full Jeltsch mod.

Thanks!

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